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DEFENCE RESEARCH ESTABLISHMENT OTTAWA
TECHNICAL NOTE 90-2

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ABSTRACT

This report discusses model order determination algorithms for autoregressive filter techniques applied to tactical radio direction finding. In particular the difficulties imposed by multipath are examined. The related issue of the determination of the number of signals for autoregressive radio direction finding techniques using the eigen decomposition method is also discussed. The performance of both model order and signal number determination methods are also evaluated using computer simulations.

RESUME

Ce rapport contient une discussion d'algorithmes de modèles auto-régressifs pour l'ordre de filtres utilisés pour la détermination de la direction d'arrivée de radios tactiques. Les difficultés crées par les trajectoires multiples recoivent une attention spéciale. On discute aussi du sujet connexe du comptage du nombre de signaux par des techniques radio de détermination de direction d'arrivée basées sur des vecteurs propres. Une simulation par ordinateur est utilisée pour évaluer les performances des méthodes de détermination de l'ordre et du nombre de signaux.

EXECUTIVE SUMMARY

This report reviews methods to determine the optimum model order of adaptive filter techniques applied to the problem of radio direction finding (DF), and the related issue of the determination of the number of signals for adaptive filter techniques using eigen decomposition techniques. This work was carried out in support of research into advanced radio direction finding techniques for land tactical applications.

For radio direction finding applications, adjusting the adaptive filter model order to a value less than the maximum (i.e. one less than the number of antennas in the sensor array) increases the ability of an adaptive filter based DF algorithm to resolve multipath signals. It is from this point of view that model order determination methods are discussed in this report.

In general, the optimum model order is not known beforehand, so a number of model order determination methods have been proposed to tackle this problem. In this report, these are classified according to the autocorrelation matrix estimator used. Three classes of estimators are considered, which are: (a) the autocorrelation method, (b) the covariance and modified covariance method, and (c) the eigen decomposition method.

The autocorrelation method is not appropriate for accurate direction finding purposes since adaptive filter DF techniques based on this method have very poor resolution. However, it is discussed in this report since the first model order determination algorithms were proposed for this method, and many of the problems encountered are common to all classes. The performance of these algorithms is quite poor, typically picking model orders that are too low.

The covariance and modified covariance methods are much superior for direction finding purposes than the autocorrelation method. It has been determined by researchers [10] both analytically and experimentally that a model order of between $N/3$ and $N/2$ is suitable, where N is the number of antenna sensors. Model orders higher than this lead to spurious estimates which severely degrade bearing accuracy.

The eigen decomposition method enhances the signal to noise ratio of the autocorrelation matrix estimated using either the covariance or modified covariance method. This enhancement suppresses spurious bearing estimates so that higher model orders can be used. Values of between $2N/3$ and $3N/4$ have been proposed.

An additional parameter required to optimally enhance the signal to noise ratio using the eigen decomposition method is the number of radio signals present. Since this problem is closely related to the model order determination problem, model determination methods have been adapted for this purpose. For widely spaced (in bearing) signals, and signal to noise ratios of 20 dB, the adapted algorithms work extremely well. Under more realistic conditons (i.e. closely spaced signals, lower signal to noise ratios, etc.) it is expected that the ability of these signal number determination algorithms to accurately determine the number of signals will degrade, but not before the bearing estimation accuracy of DF algorithms using the eigen decomposition method also degrade.

In general, methods to determine the number of signals used in conjunction with adaptive filter DF methods, where both are based on using the eigen decomposition method, appear to offer the best performance. The worst performance is provided by model order determination methods used in conjunction with DF methods where both are based on the autocorrelation method. In this case performance is very poor, and inappropriate for direction finding purposes when accurate bearing estimation is necessary.

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1.0 INTRODUCTION

Current conventional land tactical radio direction finding systems suffer from two major problems. One is the inability to accurately determine the bearing of several radio transmitters simultaneously, and the second is a degradation of performance in the presence of multipath. A considerable amount of research and development has been carried out over the last twenty years in advanced spectral estimation algorithms which has led directly to the development of superresolution radio direction finding (DF) algorithms which may be capable of dealing with these problems. In particular, adaptive filter methods (including eigen based methods) have shown promise in this area with performances approaching maximum likelihood methods [1].

The superresolution abilities of adaptive filter methods is based on matching the sensor data with an appropriate signal model. At the heart of these methods lies a system of equations, formed from the antenna sensor data, whose solution provides the signal bearings. If M signals impinge upon the sensor array (where M is less than the number of sensors), then M linearly independent equations are required to solve for the M signal bearings.

In the case where the signals are all noncoherent (transmitted by independent transmitters) M linearly independent equations may be formed from M or more sensor data vectors. In this report, a data vector is defined as a single set of measurements of all the sensor outputs taken at time instant t_m. Data vectors are assumed to be uniformly sampled in time where the sampling rate is low enough so that consecutive data vectors are not fully correlated (i.e. not scalar versions of each other).

In the case of multipath signals, consecutive data vectors remain correlated in time no matter what the sampling interval (except for the temporal noise component) so that effectively only one data vector is available. To generate the required number of linearly independent equations, the data is divided into subsets or subarrays from which the required number of linearly independent equations may be generated. This is discussed in more detail in section 1.2. In adaptive filtering terminology, the number of antenna sensors in the subarray, minus one, is called the model order. In direction finding terminology, the technique of using lower model orders to generate more linearly independent equations is called spatial averaging [2].

From the preceding discussion, it is apparent that spatial averaging is a necessity when direction finding on multipath signals, and completely unnecessary if the signals are noncoherent and a sufficient number of data vectors ($\geq M$) have been measured. Consequently, model order selection is discussed in the context of determining the bearing of the direct and indirect paths of a signal in a multipath environment.

A number of methods to determine model order and the number of signals present are evaluated in this report. In particular, they are evaluated with respect to the limitations imposed by multipath and tactical constraints which restrict the number of sensors. The effect of multipath is equivalent to DF processing using a single data vector since additional vectors are fully correlated (ignoring the noise). Consequently only single vector processing is considered in this report. To satisfy the tactical constraint the sensor arrays are limited to 16 antenna sensors. Additionally, since spatial averaging can only be used for linear sensor arrays with uniformly spaced sensors, only these type of sensor arrays will be considered.

Noise discussed in this report is assumed to be additive white Gaussian in nature and uncorrelated between sensors. For a single data vector the effects of temporal and spatial noise are indistinguishable so no distinction is made between these two types of noise.

1.1 Spectral Estimation and Radio Direction Finding

The application of spectral estimation concepts to radio direction finding is illustrated using Figure 1. A single radio signal in a noiseless environment impinges on an antenna baseline with some arbitrary direction of arrival. The amplitude and phase measured at sensor n is represented by the complex value,

$$x_n = y(t)e^{-j(n\omega_s d+\theta)} \tag{1.1}$$

where $y(t)$ represents the complex baseband modulation envelope (for all types of modulation and is assumed to remain constant for the measurement of a single vector), d is the spacing between consecutive sensors, θ is the signal phase which is a function of the distance from the transmitter to sensor 0, and

$$\omega_s = \frac{2\pi}{\lambda}\cos(\phi) \tag{1.2}$$

Here λ is the wavelength, and ϕ is the signal bearing in azimuth. (Signals with elevation angles different than 0 degrees are not considered here). Inspection of equation (1.1) reveals that x_n represents vectors of a complex sinusoid measured spatially (i.e. with respect to the sensor baseline) and that ω_s is the spatial frequency of the complex sinusoid.

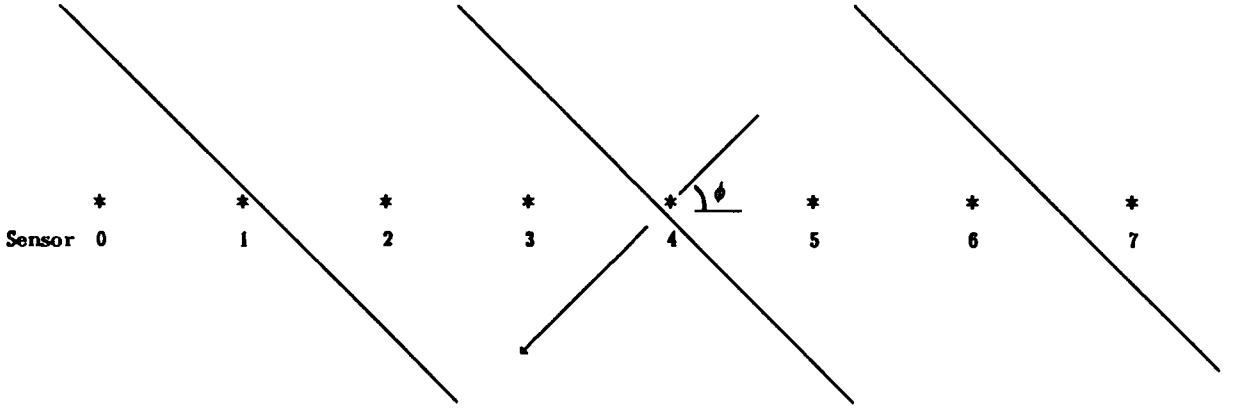

FIGURE 1: Radio direction finding sensor array

If a number of signals with different bearings impinge upon the sensor system then the spatial signal x_n is the sum of a number of complex sinusoids of different amplitudes and frequencies of the form of equation (1.1). Spectral estimation techniques can be used to resolve the various components by first generating the spatial power spectral density function. By taking advantage of equation (1.2), this spectrum is converted to a DF spectrum which gives the power density versus bearing. The location of the peaks (local maximums) in the DF spectrum are then used as the estimates for the signal bearings.

1.2 Autoregressive Model

The discussion that follows focuses on adaptive filtering spectral estimation techniques which have been adapted for the purposes of radio direction finding. The types of filters used in these applications can be classified as autoregressive (AR), moving average (MA), or autoregressive-moving average (ARMA). In general, techniques involving either MA or ARMA filters are computationally more difficult since they involve a set of nonlinear equations, and for the purposes of this report will not be considered.

Autoregressive methods are based on the idea that a uniformly sampled sequence of complex antenna sensor data, $\{x_0, x_1, x_2, \ldots, x_{N-1}\}$, (where N represents the number of sensors) can be modelled as a linear rational system driven by white noise. Mathematically,

$$x_m = -\sum_{n=1}^{p} a_n x_{m-n} + v_m, \tag{1.3}$$

where v_i is a white noise source with a variance of σ^2, and p is the model order which is chosen to be less than N. For a particular model order p, the values of the tap weight coefficients a_n are usually chosen so that the variance of the white noise v_i is minimized. Equation (1.3) also represents a linear predictor where p past data vectors are used to predict the current value x_m. In this context, v_m represents the prediction error.

Based on minimizing the noise variance, or prediction error, the optimum weighting coefficients can be found by solving the augmented normal matrix equations given by

$$\mathbf{Ra} = \sigma^2\mathbf{u}, \tag{1.4}$$

where the $(p+1)\times(p+1)$ autocorrelation matrix $\mathbf{R}$ is defined by,

$$\mathbf{R} = \begin{bmatrix} E\{x_0 x_0^*\}, & E\{x_0 x_1^*\}, & E\{x_0 x_2^*\}, \ldots, & E\{x_0 x_p^*\} \\ E\{x_1 x_0^*\}, & E\{x_1 x_1^*\}, & E\{x_1 x_2^*\}, \ldots, & E\{x_1 x_p^*\} \\ E\{x_2 x_0^*\}, & E\{x_2 x_1^*\}, & E\{x_2 x_2^*\}, \ldots, & E\{x_2 x_p^*\} \\ \vdots & \vdots & \vdots & \vdots \\ E\{x_p x_0^*\}, & E\{x_p x_1^*\} & E\{x_p x_2^*\} & \ldots, E\{x_p x_p^*\} \end{bmatrix}, \tag{1.5}$$

the augmented tap weight vector $\mathbf{a}$ is defined by,

$$\mathbf{a} = \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix}, \tag{1.6}$$

and the vector $\mathbf{u}$ is defined by,

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{1.7}$$

Using autoregressive filter techniques, an autoregressive DF estimator can be defined by [3]

$$S(\phi) = \frac{\sigma^2}{|\mathbf{a}^H \mathbf{e}|^2}, \tag{1.8}$$

where $\mathbf{e}$ is the $p+1$ element steering vector defined by,

$$\mathbf{e} = \begin{bmatrix} 1 \\ e^{+j\omega_s d} \\ e^{+j\omega_s 2d} \\ \vdots \\ e^{+j\omega_s pd} \end{bmatrix}. \tag{1.9}$$

and ω_s was defined previously by equation (1.2).

1.3 The Estimated Autocorrelation Matrix

To this point, the calculation of the tap weight filter coefficients, and the formation of the spatial spectral density function (equation (1.8)) using these coefficients has been discussed. The assumption was made that the optimum model order p was known. In the case where the true autocorrelation matrix is known, the higher the order of p, the better the resolution of the AR spectral estimator. Consequently, when the true autocorrelation matrix is known, the best value of p is equal to N-1.

In most practical estimation problems the true autocorrelation matrix is unknown and must be estimated from the data. The estimated autocorrelation matrix can be defined by the equation

$$\hat{\mathbf{R}} = \sum_i \mathbf{x}_i \mathbf{x}_i^H + \sum_j \mathbf{y}_j \mathbf{y}_j^H \tag{1.10}$$

where $\mathbf{x}_i$ represents a $p+1$ length column vector containing the sensor data in the forwards direction. That is,

$$\mathbf{x}_i = \begin{bmatrix} x_i \\ x_{i+1} \\ x_{i+2} \\ \vdots \\ x_{i+p} \end{bmatrix}, \tag{1.11}$$

and $\mathbf{y}$ represents a $p+1$ length column vector containing the conjugated sensor data in the reverse direction. That is,

$$\mathbf{y}_j^* = \begin{bmatrix} x_{j+p} \\ x_{j+p-1} \\ x_{j+p-2} \\ \vdots \\ x_j \end{bmatrix}. \tag{1.12}$$

The range of the indices i and j have purposely been left unspecified since different methods use different ranges.

2.0 MODEL ORDER DETERMINATION FOR THE AUTOCORRELATION METHOD

The autocorrelation method of estimating the autocorrelation matrix can be defined from equation (1.10) as

$$\hat{\mathbf{R}} = \sum_{i=-p}^{N-1} \mathbf{x}_i \mathbf{x}_i^H \tag{2.1}$$

Since this expression involves unknown data (x_1, x_2, etc., and x_N, x_{N+1}, etc.), the unknown data is assumed to be zero. In other words, the data is pre- and postwindowed. Windowing results in a loss of resolution which becomes progressively worse as the number of sensors, N, decreases. As a result, the autocorrelation method is generally not favoured for spectral estimation problems involving short data records (such as direction finding), however, it is mentioned here since some of the earliest model order determination algorithms were evaluated using this approach.

Figure 2 is an example of the effect that model order selection has on the AR spectral estimate where the autocorrelation matrix described by equation (1.10) was used. In this example a radio direction finding system using a linear 16 element sensor array with 0.5λ spacing was simulated. The actual signals had unit power and bearings of 40 and 100 degrees respectively. The noise variance was 0.01.

Inspection of Figure 2 reveals that when the model order is less than the number of signals (in this case $p<2$), the resolution of the spectral estimator is seriously degraded. In this case there are fewer filter coefficients than there are signals.

When the model order becomes too large (in this case $p>4$) the spectral peaks corresponding to the signal locations become corrupted and spurious peaks begin to appear. In this case there are more filter coefficients than there are signals. This leads to better resolution, however, the extra filter coefficients, which model the noise, occasionally give rise to spurious estimates as seen in this example.

Another problem is that for successively higher model orders the elements of the autocorrelation matrix are formed from fewer and fewer data values. This leads to greater variance in the autocorrelation matrix estimate and in turn leads to greater variance in the spectral estimate.

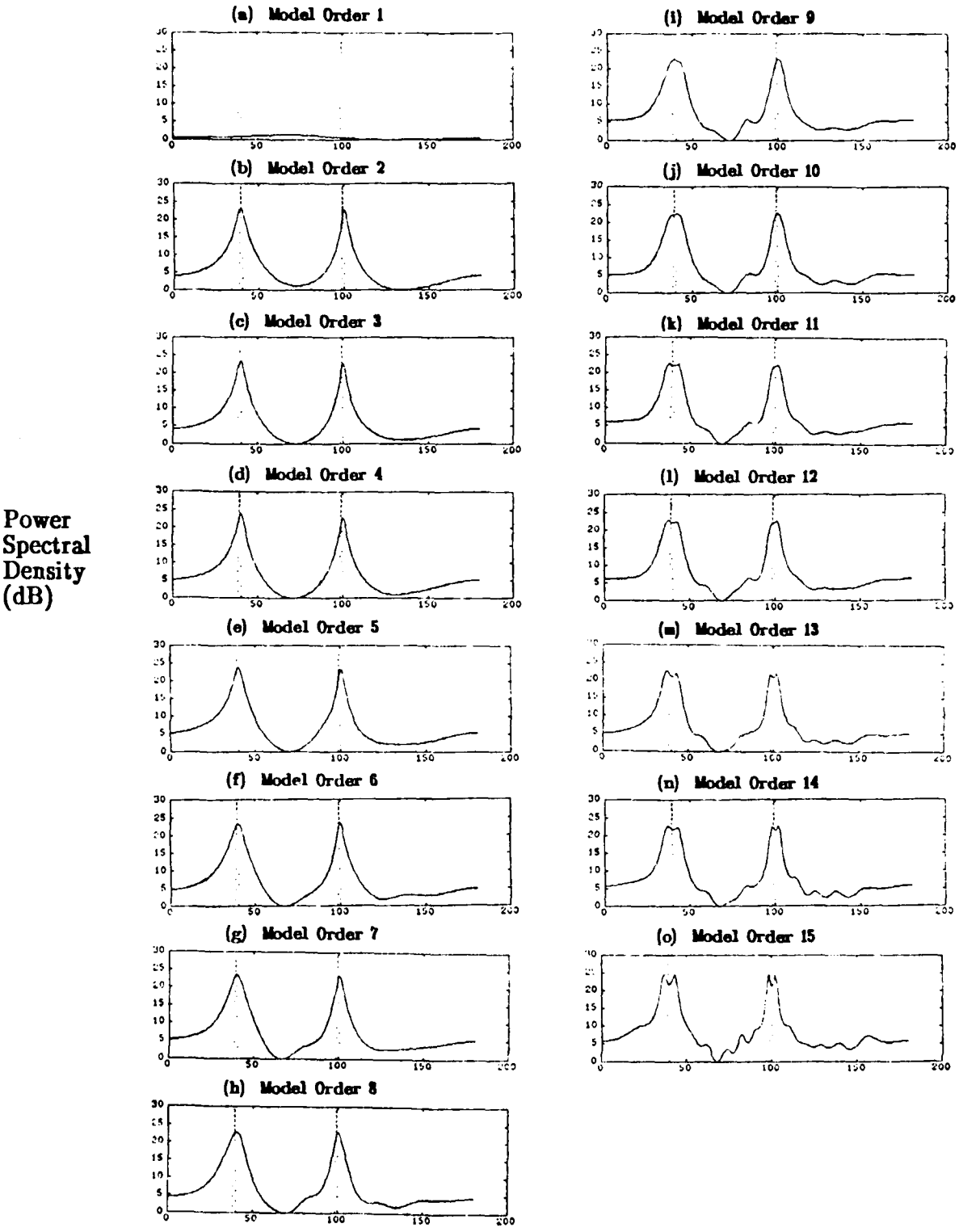

FIGURE 2: Effect of the choice of AR model order on the estimated DF spectrum. Actual signal bearings are shown by the dashed lines. Signal powers ($|\text{amplitude}|^2$) were 1.0 and the noise variance was 0.01 (i.e. signal to noise ratio of 20 dB).

To quantify the effect of model order selection on DF accuracy, 100 DF trials were simulated for a two signal case (M=2) and a five signal case (M=5) using the same DF sensor system described for Figure 2 (i.e. 16 elements). The results are shown in Figure 3. For each trial the signal powers were set to 1, the signal phases were randomly chosen between 0 and 2π, and the signal bearings were set to 40 and 100 degrees for the two signal case, and 40, 70, 100, 140, and 160 for the five signal case. The noise variance was set at 0.01 (SNR = 20 dB). Bearing errors were calculated by locating the M largest peaks in the DF spectrum and subtracting these bearings from the true bearings. In cases where there were less than M peaks in the DF spectrum, such as when the model order $p < M$, the error was calculated as the difference between the corresponding estimated bearing and the nearest true bearing. The fact that some bearings were completely missed this way was not taken into account in the results. The accuracy measurements shown in Figure 3 were computed as the standard deviation of the bearing error for a particular model order.

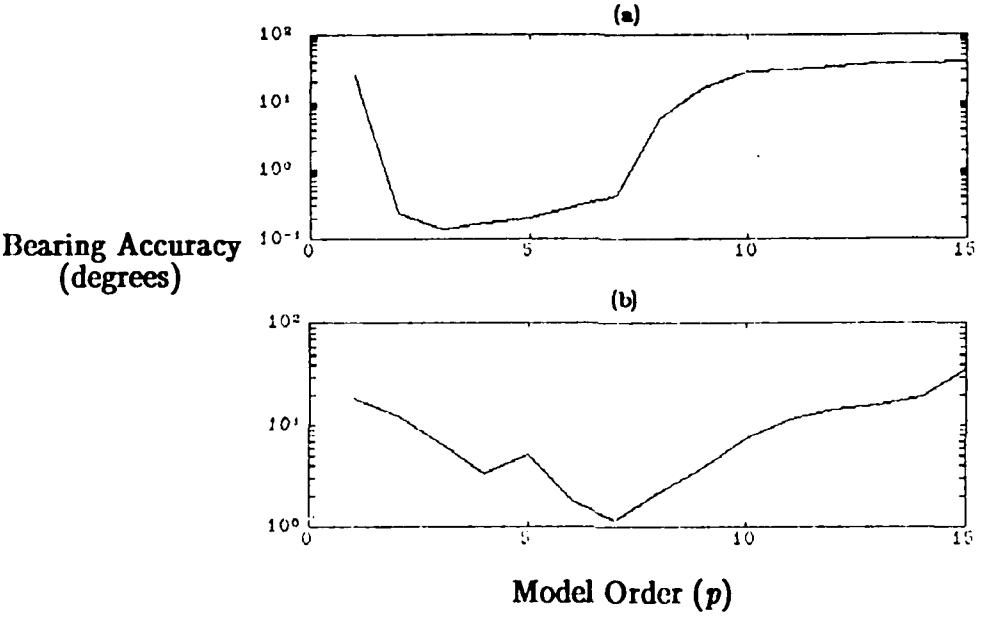

FIGURE 3: Bearing estimation accuracy versus model order for the case of equi-amplitude signals (SNR = 20 dB) with bearings of: (a) 40 and 100 degrees, and (b) 40, 70, 100, 140, and 160 degrees.

From the results shown in Figure 3, it is clear that the optimum model order is equal to or slightly greater than the number of signals present. The large error that occurs for model order selections that are too low or too high supports the observations made in Figure 2. It should be pointed out that Figure 3 is highly contrived (compared to what might be expected in a true multipath environment) since the parameters of the true signals used in Figure 3 were chosen to be within the resolution capabilities of the AR spectral estimator, i.e. bearings were widely spaced, and signal powers were reasonably strong compared to the noise power. However, in cases where the AR estimator is unable to resolve two or more signals, the accuracy of the estimate degrades rapidly to the point that the estimates for any model order are not much more accurate than random guesses. In general, Figures 2 and 3 indicate the need for some method to choose the correct model order.

Algorithms designed to determine the best model order take advantage of two

properties of AR filters. The first property is that as the model order is increased from 1 to N-1, the rate of decrease of the prediction error variance σ^2 (see equation (1.4)), is greater when the model order is less than the number of signals than compared to when the model order is greater than the number of signals. The second property is that as the model order increases, the variance in the estimates of the AR tap weight coefficients increases.

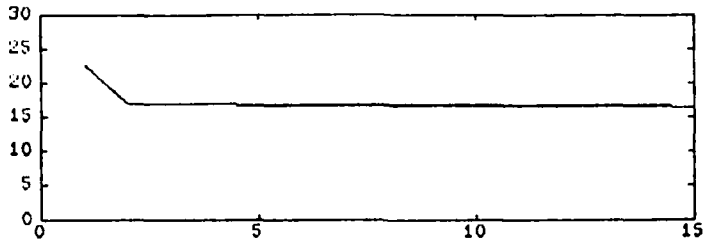

FIGURE 4: Prediction error variance versus model order for the example shown in Figure 2.

Figure 4 demonstrates the first property where the "kink" in the prediction error variance curve corresponds to the number of signals, which in this case was 2. Although one might be tempted to base model order determination on locating this kink, the problem becomes far more difficult when closely spaced signals in bearing, lower signal to noise ratios and signals of different powers are considered, as illustrated in Figure 5. However, by taking advantage of the second property, the optimum model order can be estimated by comparing the decreasing prediction error (as shown in Figure 4) versus increasing bearing estimation variance as the model order is increased. Most of the model order determination algorithms discussed in this report are based on this idea.

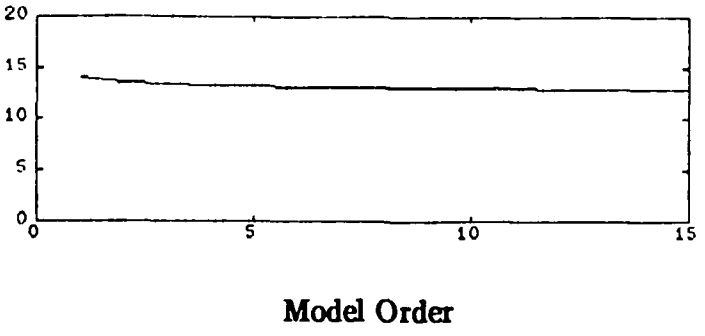

FIGURE 5: Prediction error variance versus model order for signal bearings of 40 and 70 degrees, and powers of 1.0 and 0.1 units.

One of the earliest approaches, called the Final Prediction Error algorithm (FPE) [4] was proposed by Akaike in 1969. It is given by,

$$\mathrm{FPE}(p) = \sigma_p^2 \left[\frac{N + (p+1)}{N - (p+1)} \right] \tag{2.2}$$

where σ_p^2 is the variance of the prediction error for the AR model order p. It is assumed that the sample mean has been subtracted from the data (this assumption also applies to the rest of the algorithms discussed in section 2.0). The minimum of the above function gives the estimate of the optimum model order.

In 1974 Akaike proposed another model order selection algorithm called the Akaike Information Criterion (AIC) [4] based on a maximum likelihood approach. Assuming the process has Gaussian statistics, the algorithm is given by

$$\mathrm{AIC}(p) = N \ln(\sigma_p^2) + 2p \tag{2.3}$$

The model order is again selected as the value which minimizes this function.

Statistically, it was determined that the AIC method was inconsistent since the probability of error in choosing the correct model order does not approach zero as the number of data values N approaches infinity. In response to this, Rissanen developed a variant algorithm in 1983 which is statistically consistent for large N, called the Minimum Description Length (MDL) [5]. Again assuming Gaussian statistics, the model order is determined by minimizing the following function,

$$\mathrm{MDL}(p) = N \ln(\sigma_p^2) + 2p \ln(N) \tag{2.4}$$

At the same time that Akaike introduced the AIC algorithm, Parzen introduced an algorithm he called the Criterion Autoregressive Transfer function (CAT) [6]. The best estimate of the optimum model order is determined by minimizing the function

$$\mathrm{CAT}(p) = \left(\frac{1}{N} \sum_{j=1}^{p} \nu_j^{-1}\right) + \nu_p^{-1} \tag{2.5}$$

where

$$\nu_j = [N/(N-j)]\ \sigma_j^2 \tag{2.6}$$

Figure 6 provides two examples of the performance of these algorithms for a single sensor data vector (again using a 16 element array with the same signal and noise parameters used in Figure 3). In these two examples, all the algorithms were successful in choosing the optimum model order (i.e. equal to, or slightly greater than the number of signals).

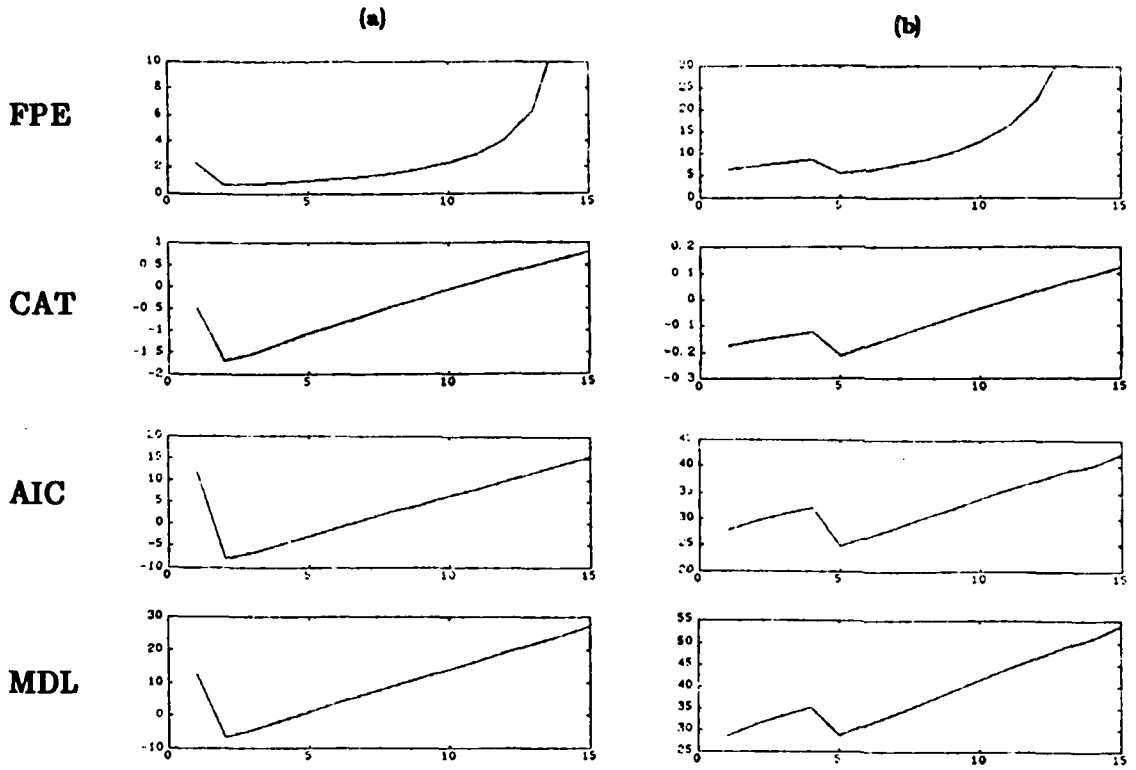

FIGURE 6: Model order determination algorithms for a (a) two signal case, and (b) a five signal case

In general, the derivation of the algorithms described in this section depend on approximations which are true for large samples (i.e. N large) but do not necessarily hold in the case of short data records. Researchers have found, that for the case of short data records involving sinusoids in noise, none of the algorithms work well [7] and in general tend to pick a model order that is too low. This conclusion is also supported by the results shown Figure 7 which gives a histogram of the predicted model orders for each algorithm for 100 trials of a 5 signal environment (the same sensor data that was used in Figure 3(b)).

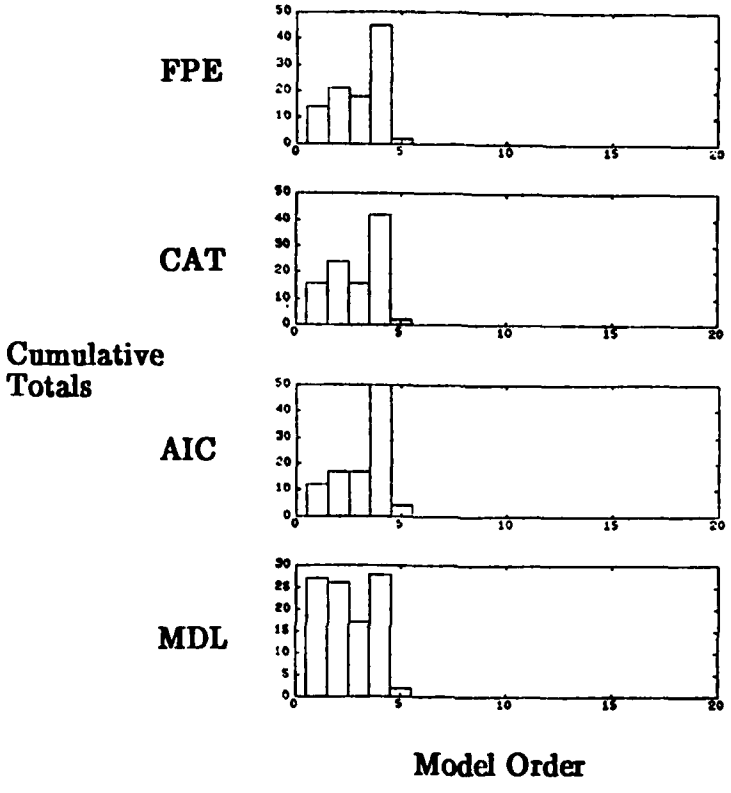

FIGURE 7: Performance of the model order determination algorithms using the sensor data from Figure 3(b), i.e. 100 trials of a 16 element system in a five signal environment.

3.0 MODEL ORDER DETERMINATION FOR THE COVARIANCE AND MODIFIED COVARIANCE METHOD

The covariance method of estimating the autocorrelation matrix uses only known data, thus avoiding the use of windowing which degrades the resolution of the spectral estimator. Based on equation (1.10) the covariance method can be defined as,

$$\hat{\mathbf{R}} = \sum_{i=0}^{N-p-1} \mathbf{x}_i \mathbf{x}_i^H. \tag{3.1}$$

An important feature of the autocorrelation matrix estimated in this manner is that the rank of the matrix is the lesser of N-p (the number of linear independent data vectors used), or, p+1, (the length of the data vector).

The modified covariance method takes advantage of the relationship between the forward data vector $\mathbf{x}_i$ and the conjugated reverse data vector $\mathbf{y}_i$ given by,

$$E\{\mathbf{x}_i \mathbf{x}_i^H\} = E\{\mathbf{y}_i \mathbf{y}_i^H\}. \tag{3.2}$$

This is used to generate an improved autocorrelation matrix estimate which can be defined as

$$\hat{\mathbf{R}} = \sum_{i=0}^{N-p-1} \mathbf{x}_i \mathbf{x}_i^H + \sum_{i=0}^{N-p-1} \mathbf{y}_i \mathbf{y}_i^H. \tag{3.3}$$

The advantage of this method, compared to the covariance method, is that each autocorrelation matrix element is produced from twice as many data values which results in less variance in the estimates. Additionally since twice as many linearly independent data vectors are used, the rank of the matrix is the lesser of $2(N\text{-}p)$ or p+1. Based on rank considerations, the maximum number of signal bearings that may be solved using the modified covariance method is $2N/3$ as compared to $N/2$ for the covariance method. Given these advantages, and since very little extra processing is involved, the modified covariance method is the more preferred method for direction finding type problems. Most of the following discussion centers on the modified covariance method. However, the remarks made concerning this method may readily be extended to the covariance method by taking the differences discussed above into account.

Figure 8 illustrates the effect of changing the model order on the spectral estimates when the modified covariance method is used (the sensor data in this example was the same used in Figure 2). As in the autocorrelation method, the best results occur for model orders equal to or slightly greater than the number of signals present.

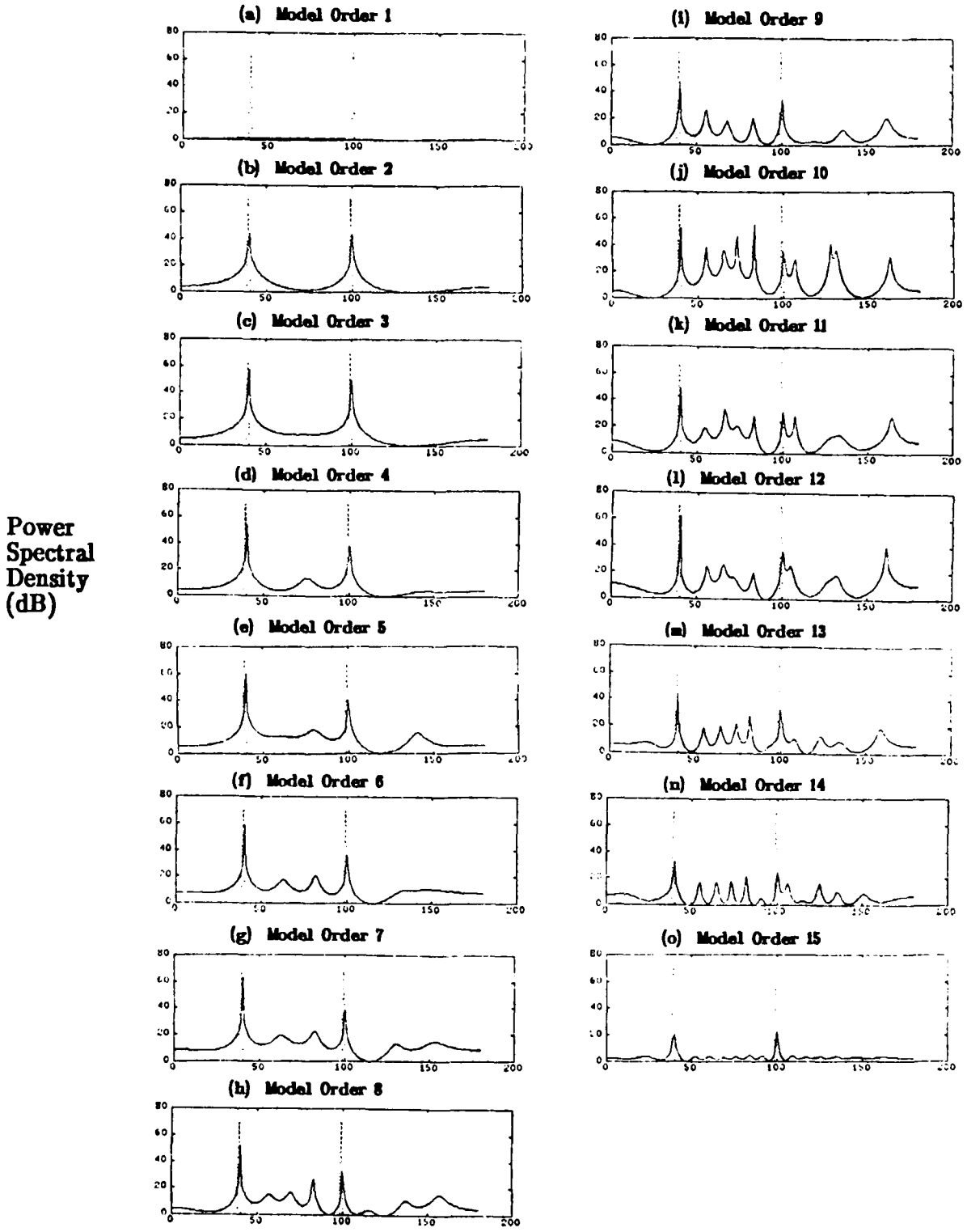

FIGURE 8: Effect of the choice of AR model order on the estimated DF spectrum. Actual signal bearings are shown by the dashed lines. Signal powers were 1.0 and the noise variance was 0.01 (i.e. signal to noise ratio of 20 dB).

Figure 9 shows the effect of changing model order on the accuracy of the bearing estimates using the modified covariance method. The sensor data, and method of computing the sensor data was described previously in section 2.0 for Figure 3. An examination of Figure 9(b) shows that this plot can be broken down into 5 separate regions. In the first region, $p<5$, the model order is less than the number of signals present so that it is not possible to determine all the signal bearings. As in the autocorrelation method, this results in poor estimates of the spatial frequency spectrum.

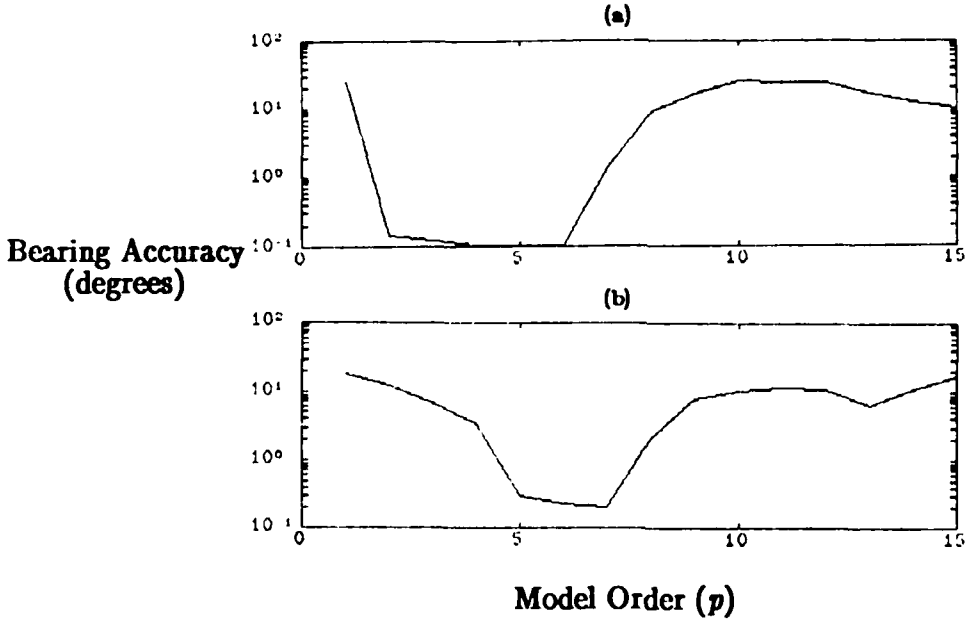

FIGURE 9: Bearing estimation accuracy versus model order for the case of equi-amplitude signals (SNR = 20 dB) with bearings of: (a) 40 and 100 degrees, and (b) 40, 70, 100, 140, and 160 degrees.

In the second region, $5\leq p\leq 7$, the model order is sufficiently high that it is now possible to determine all the signal bearings, but not high enough that spurious estimates occur. In comparing the estimation accuracy in this region to the rest of the plot, it is clear that this region represents the optimum order for the AR spectral estimator.

In the third region, $8\leq p\leq 10$, spurious estimates occur which degrade the accuracy of the bearing estimates (i.e. if a spurious peak in the DF spectrum is larger than any of the signal peaks, it is then mistakenly assumed to be a signal peak). The error introduced by the spurious estimates increases as the number of extra model coefficients (p-M) increases. Additionally as the order increases, the variance in the estimates increases as well due to the decreasing number of data values used to form each element of the autocorrelation matrix.

In the fourth region, $11\leq p\leq 13$, although the model order is increasing, the rank of the autocorrelation matrix decreases according to the function $2(N-p)$. As a result, the filter tap weight coefficients can no longer be uniquely determined. An appropriate choice for the tap weight coefficient vector (defined by equation (1.6)) is the vector which has the

minimum norm (where the vector norm is given by $\mathbf{a}^H\mathbf{a}$) of all possible solutions. This vector is unique and has the desirable property that spurious estimates are suppressed [8]. Using the tap weight coefficients generated in this manner, the reduced rank effectively reduces the number of signals (including spurious signals) that may be uniquely determined. In consequence, as the order increases, the rank decreases, and the number of spurious signals and resultant bearing error variance decreases.

A special case occurs when the rank of the matrix given by $2(N\text{-}p)$, equals the number of signals M (such as for p=15 in Figure 9(a)). This is called the Kumaresan-Prony case [9] and is of special interest since only the actual signal bearings can be uniquely determined i.e. spurious signals do not occur. Choosing the model order corresponding to this special case is also equivalent to using noise suppression techniques discussed in section 4.0 without the penalty of additional processing. The difficulty is that knowledge of the exact number of signals is required, an issue which is also discussed in section 4.0.

In the fifth region, $p>13$, the rank of the autocorrelation matrix decreases below the minimum required value (in this case 5) and so suffers the same problems as those of the first region, plus in addition, the effect of increasing variance in the autocorrelation matrix estimate.

To determine the correct model order, the algorithms discussed in section 2.0 could be used. However, as discussed previously, these algorithms have not been found to work well for short data segments. Lang and McClellan [10] showed analytically that a model order selection of about $N/3$ provides good results for both the covariance and modified covariance methods. This seems reasonable in view of the results shown in Figure 9, assuming the number of signals $M<N/3$, otherwise errors will occur since there would be an insufficient number of tap weight coefficients to estimate all the signal bearings.

4.0 MODEL ORDER AND SIGNAL NUMBER DETERMINATION USING THE EIGEN DECOMPOSITION METHOD

Improved AR spectral estimates can be made by taking advantage of eigen decomposition techniques to generate a better estimate of the noiseless autocorrelation matrix, and improve the spectral estimates. A summary of this method is given in the following paragraphs.

For the case of signals in white noise, the autocorrelation matrix can be represented as

$$\mathbf{R} = \sum_{k=0}^{M-1} \lambda_k \mathbf{v}_k \mathbf{v}_k^H + \eta^2 \mathbf{I}, \qquad (4.1)$$

where λ_k represents the non-zero eigenvalues of the noiseless autocorrelation matrix and are ordered from largest (λ_0) to smallest, $\mathbf{v}_k$ represents the corresponding eigenvectors and η^2 represents the noise variance of the sensor noise. The first term represents the signal correlation matrix where it is assumed that $M \leq p$ and the second term represents the noise correlation matrix. Ideally, knowing the signal correlation matrix would allow the calculation of the exact signal bearings.

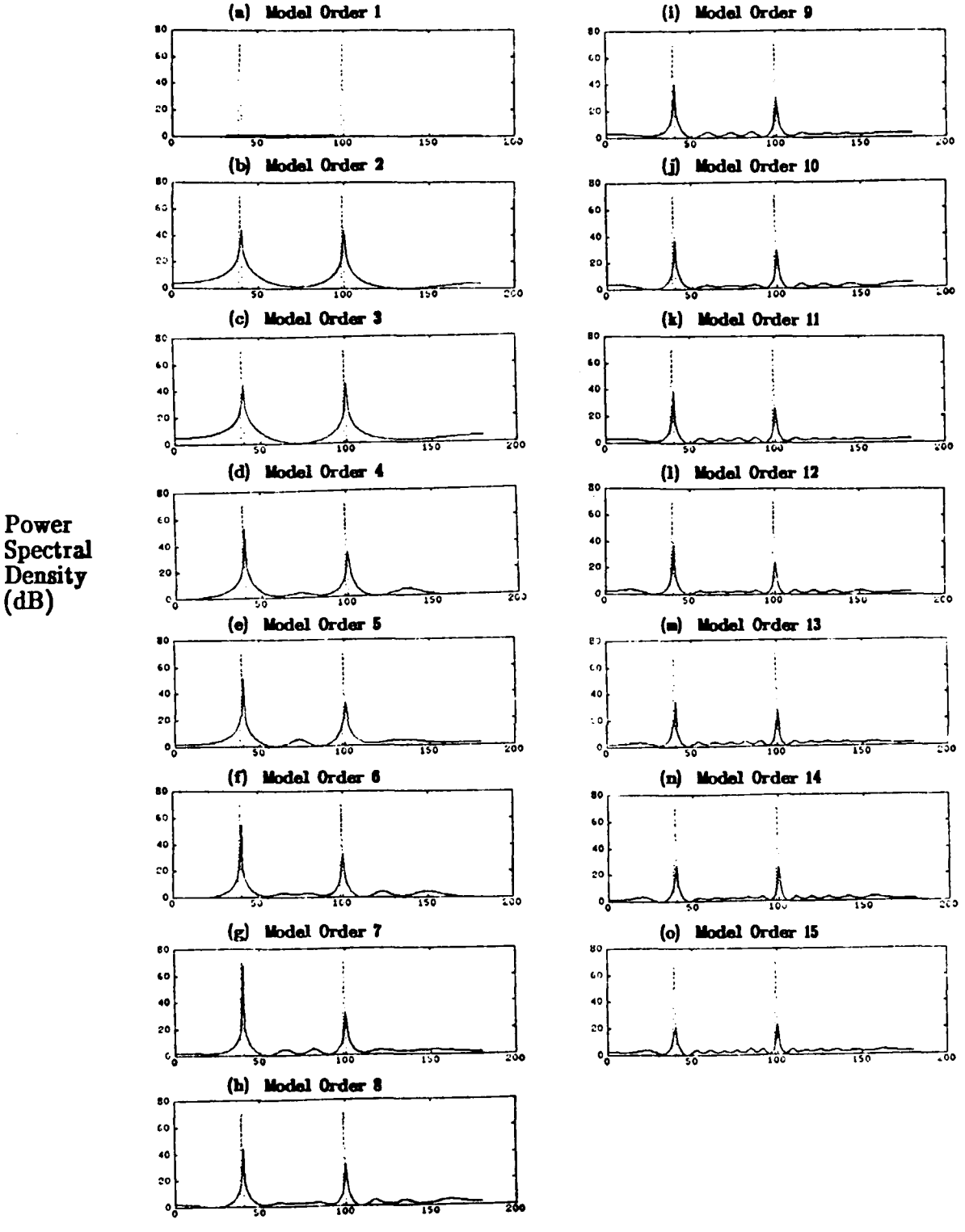

FIGURE 10: Effect of the choice of AR model order on the estimated DF spectrum. Actual signal bearings are shown by the dashed lines. Signal powers were 1.0 and the noise variance was 0.01 (i.e. signal to noise ratio of 20 dB).

The actual eigen decomposition of the autocorrelation matrix is given by,

$$\mathbf{R} = \sum_{k=0}^{M-1}(\lambda_k+\eta^2)\mathbf{v}_k\mathbf{v}_k^H + \sum_{k=M}^{N-1}\eta^2\mathbf{v}_k\mathbf{v}_k^H \qquad (4.2)$$

where $\lambda_k+\eta^2$ represents the M largest eigenvalues of **R** and η^2 represents the N-M+1 smallest eigenvalues of **R**. The first summation term in the above expression is called the signal subspace since it is spanned by the signal vectors (plus noise) while the second summation term is called the noise subspace since it is spanned only by noise vectors.

The above results are only approximately true in the case of the estimated autocorrelation matrix, and only apply if either the covariance or modified covariance methods are used. (The data windowing used in the autocorrelation method spreads some of the signal power into the noise eigenvalues making it difficult to separate the signal and noise subspaces for small data vectors). However, by performing the eigen decomposition on the estimated autocorrelation matrix and then using its signal subspace equivalent (i.e. the smallest N-M+1 eigenvalues are set to zero) improved signal bearing estimates result.

Since the deletion of the noise subspace results in a reduced rank autocorrelation matrix, there are an infinite number of solutions for the tap weight coefficient vector. For the same reasons discussed previously in section 3.0, the solution vector is chosen as the one with the minimum norm. The improvement in the DF estimates - as manifested by the suppression in spurious peaks - using these techniques and shown in Figure 10 is obvious when compared to Figure 8.

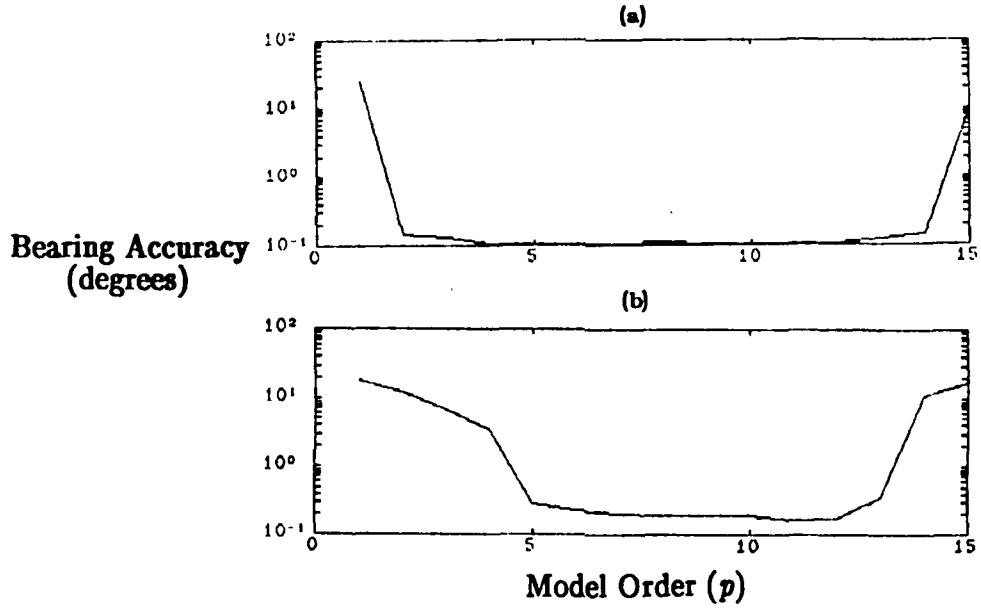

FIGURE 11: Bearing estimation accuracy versus model order for the case of equi-amplitude signals (SNR = 20 dB) with bearings of: (a) 40 and 100 degrees, and (b) 40, 70, 100, 140, and 160 degrees.

Figure 11 shows the effect of changing model order on the accuracy of the spectral estimator for the identical data set as was used in Figures 3 and 9. In this case the effects of noise have been significantly reduced which has the effect of relaxing the restriction on the choice of the optimum model order. Kumaresan and Tufts have experimentally determined that a model order of $p=(3/4)N$ is a reasonable choice [8] which is supported by the results shown in Figure 11. However, in using the modified covariance method, a choice of $p=(2/3)N$ might be more suitable since this results in the maximum rank of the autocorrelation matrix, and correspondingly, the maximum number of signals that may be estimated, without any significant loss in resolution.

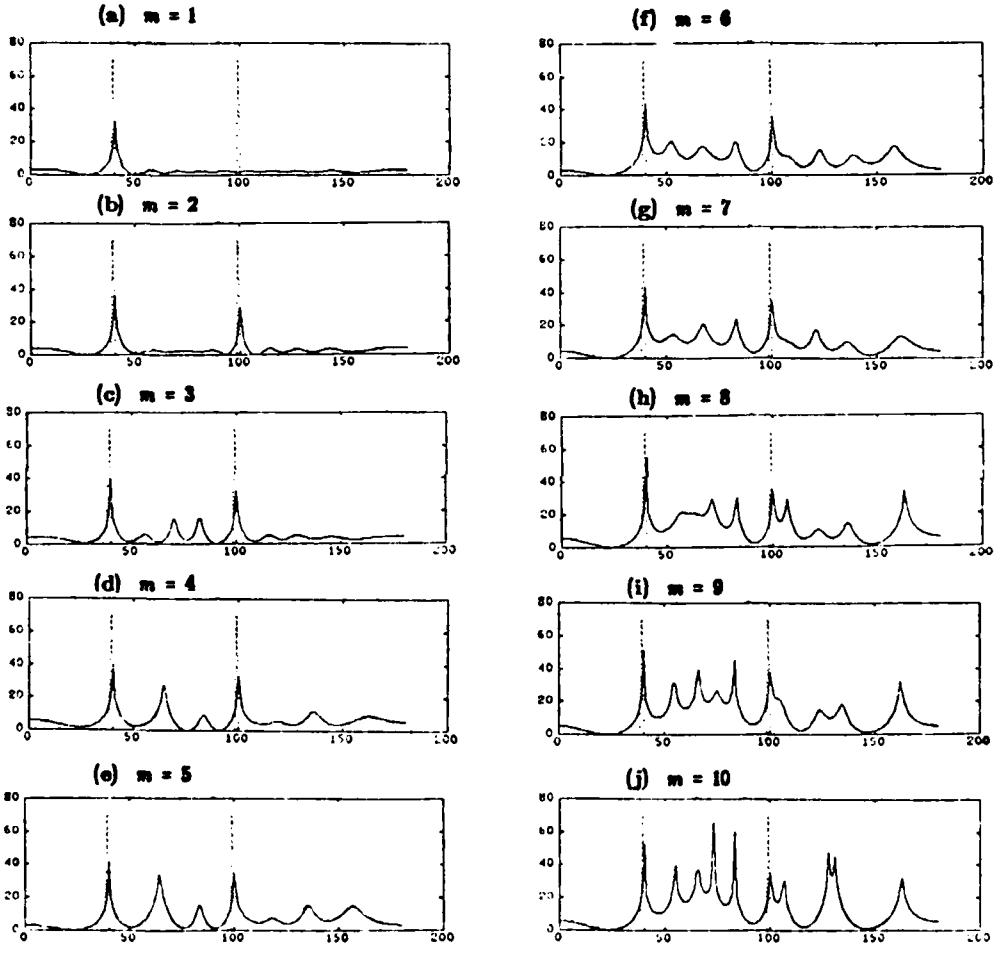

FIGURE 12: Effect of the choice of signal number, m, on the estimated DF spectrum. AR filter order $p=10$ (see Figure 10(j)).

Up to this point it has been assumed that the number of signals was known which would allow one to make the division between the noise and signal subspaces in equation 5.2. In practice this is an unrealistic assumption and some method is required to determine this value since spurious estimates can result when the incorrect value is chosen as shown in Figure 12. One method that has been proposed is to examine the eigenvalues to try and determine the division between the larger signal subspace eigenvalues and smaller noise subspace eigenvalues (see Figure 13). This is analogous to examining the forward prediction error variance to determine model order as discussed in section 3.0, and the problems in doing the determination this way are identical.

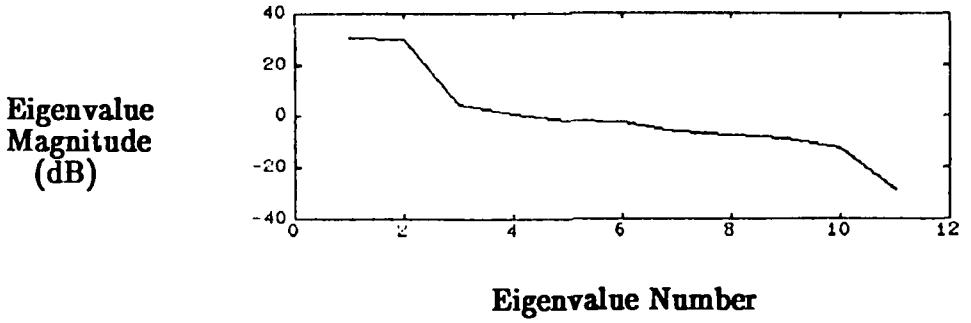

FIGURE 13: Eigenvalues of the autocorrelation matrix used in Figure 12

Using a similar approach as discussed in section 3.0, Kailath and Wax have reformulated the AIC and MDL algorithms based on using the eigenvalues of the autocorrelation matrix [11]. The results are given by,

$$\mathrm{AIC}(m) = N\,(p - m)\,\ln\left[\frac{\frac{1}{p-m}\sum_{i=m+1}^{p}\lambda_i}{\prod_{i=m+1}^{p}\lambda_i^{\,1/(p-m)}}\right] + m\,(2p - m), \qquad (4.3)$$

and,

$$\mathrm{MDL}(m) = N\,(p - m)\,\ln\left[\frac{\frac{1}{p-m}\sum_{i=m+1}^{p}\lambda_i}{\prod_{i=m+1}^{p}\lambda_i^{\,1/(p-m)}}\right] + \frac{1}{2}\,m\,(2p - m)\,\ln(N) \qquad (4.4)$$

The optimum signal number m is found by minimizing the above functions. The values λ_i are the eigenvalues of the $p \times p$ autocorrelation matrix which is formed by deleting the first

row and first column of the $(p+1)\times(p+1)$ estimated autocorrelation matrix (formed using either the covariance or modified covariance method).

Figure 14 shows two examples of how both the AIC and MDL algorithms perform for the same data used in Figure 6 (section 2.0).

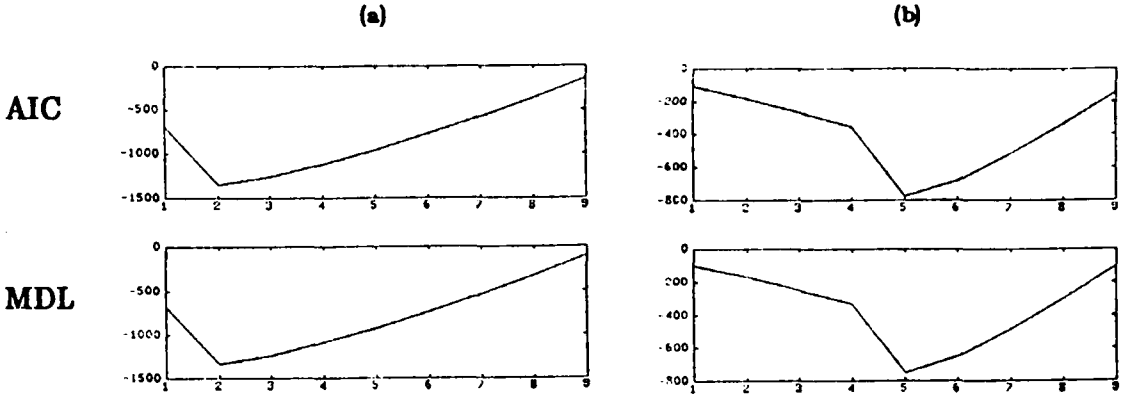

FIGURE 14: Signal number determination algorithms for a (a) two signal case, and (b) a five signal case

At the present time, little is available in the open literature describing the performance of these algorithms. To provide some idea of their performance both the AIC and MDL algorithms were tested against sensor data simulated for a 16 element DF array over 100 trials (the number of signals was 5). The method of generating the data was identical to that described for Figure 3(b) and Figure 7 in section 2.0. The results are shown in Figure 15. In this particular example both the AIC and MDL algorithms were perfect.

To test these algorithms against a more difficult signal environment, the simulations were re-run with signal powers randomly chosen to be between 10 and 40 dB greater than the noise power, signal phases randomly chosen between 0 and 2π, and signal bearings randomly chosen between 0 and 180 degrees. Figure 16 (a) shows the results. Under these conditons the performance of both algorithms was severely degraded with the correct number of signals (5) being determined less than 50% of the time. However, in examining the cases when the AIC or MDL algorithm failed to correctly determine the number of signals, it was determined that in most of these cases it was not possible to accurately estimate all signal bearings from the DF spectrum. Consequently the trials for Figure 16(a) were rerun, except trials where any of the five estimated bearings deviated from the true bearing by more than 5 degrees were discarded. Figure 16(b) shows the results.

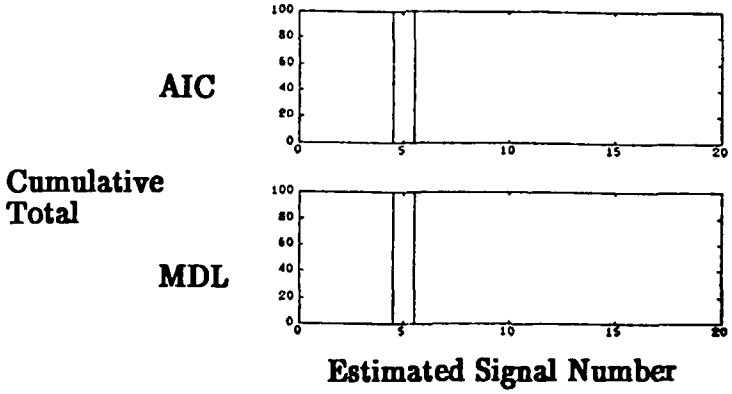

FIGURE 15: Performance of the model order determination algorithms using the sensor data from Figure 3(b), i.e. 100 trials of a 16 element system in a five signal environment.

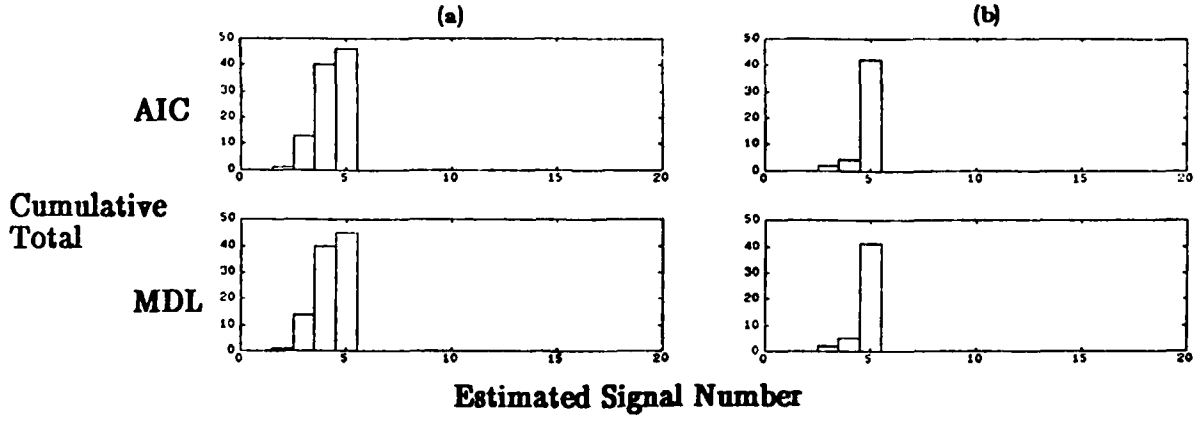

FIGURE 16 Performance of the model order determination algorithms as in Figure 15 except (a) signal powers, phases, and bearings were randomly chosen, (b) same as (a) except trials where estimated bearings were > 5 degrees were ignored.

5.0 SUMMARY AND CONCLUSIONS

In this report, methods to determine the optimum model order of adaptive filter techniques (in particular autoregressive filters) applied to the problem of radio direction finding were evaluated. In filter terms model order is the number of filter stages. In direction finding (DF) terms, the model order determines the antenna subarray size for spatial smoothing. In the case of direction finding techniques using the eigen decomposition method, the related problem of determination of the number of signals was also considered.

Spatial smoothing is a technique which is used to improve direction finding accuracy in the presence of multipath by using antenna subarrays (smaller than the full array) to decorrelate the various multipath signal components. For the noncoherent signal case (uncorrelated signals transmitted by separate sources), this technique would generally not be required since the signals can be decorrelated in time, and consequently the full array is used (which corresponds to a model order of N-1 where N is the number of antennas in the DF array).

In a multipath environment, the optimum model order is generally not known before hand, so a number of model order determination methods have been proposed to tackle this problem. In this report, these were classified according to the autocorrelation matrix estimator used. Three classes of estimators were considered. They were: (a) the autocorrelation method, (b) the covariance and modified covariance method, and (c) the eigen decomposition method.

The autocorrelation method is not appropriate for accurate direction finding purposes since adaptive filter DF techniques based on this method have very poor resolution. However, it was discussed since the first model order determination algorithms were proposed for this method, and many of the problems encountered are common to all classes. Four algorithms were considered, namely, Forward Prediction Error (FPE), Criterion Autoregressive Transfer function (CAT), Akaike Information Criterion (AIC), and Minimum Description Length (MDL). The performance of these algorithms is quite poor, typically picking model orders that are too low.

The covariance and modified covariance methods are much superior for direction finding purposes than the autocorrelation method. It has been determined both analytically and experimentally that a model order of between $N/3$ and $N/2$ is suitable. At higher model orders spurious peaks begin to appear in the DF spectrum. This leads to a severe degradation in the bearing accuracy since these spurious peaks are often indistinguishable from signal peaks, and consequently erroneous bearing estimates are made using these peaks.

The eigen decomposition method enhances the signal to noise ratio of the autocorrelation matrix estimated using either the covariance or modified covariance method. This enhancement suppresses spurious bearing estimates so that higher model orders can be used. Values of between $2N/3$ and $3N/4$ have been proposed.

One difficulty with using the eigen decomposition method is that the number of signals is required. In practice, this value is generally unknown. This problem is closely related to the model order determination problem, and as a consequence, both the AIC and MDL model order determination algorithms have been reformulated for this problem. In relative terms, the performance of these algorithms is much superior than the corresponding algorithms tested for the autocorrelation method. From the results of

simulations performed for this report, it is expected that the performance of these algorithms will degrade under realistic conditions (i.e. closely spaced signals in bearing, low signal to noise ratios, etc.), but that this will occur at the same time as the accuracy of eigen based DF algorithms also degrade.

In general, signal determination methods used in conjunction with DF methods, where both are based on using the eigen decomposition method, appear to offer the best performance when the accuracy of the DF bearing estimates is considered. Model order determination methods based on the autocorrelation method were found to perform poorly and are considered inappropriate for DF purposes.

6.0 REFERENCES

1. Tufts, D., and Kumaresan, R., "Estimation of Frequencies of Multiple Sinusoids: Making Linear Prediction Perform Like Maximum Likelihood", *Proceedings of the IEEE*, VOL. 70, No. 9, September 1982.

2. Shan, T., Wax, M., and Kailath, T., "On Spatial Smoothing for Direction-of-Arrival Estimation of Coherent Signals", *IEEE Trans. on Acoustics, Speech, and Signal Processing*, Vol. ASSP-33, No. 4, August 1985.

3. Marple, S., *Digital Spectral Analysis with Applications*, Alan Oppenheim, editor, Prentice-Hall, Englewood Cliffs, New Jersey, Chapter 6, 1987.

4. Akaike, H., "A New Look at the Statistical Model Identification", *IEEE Trans. on Automatic Control*, vol AC-19, No. 6, pp. 716-723 December 1974.

5. Rissanen, J., "A Universal Prior for the Integers and Estimation by Minimum Description Length", *Ann. Stat.*, vol.11, pp. 417-431, 1983.

6. Parzen, P., "Some Recent Advances in Time Series Modelling", *IEEE Trans. on Automatic Control*, vol AC-19, No. 6, pp. 723-730, December 1974.

7. Ulrych, T.J., and Clayton, R.W., "Time Series Modeling and Maximum Entropy", *Phys. Earth Planet. Inter.*, vol. 12, pp. 188-200, August 1976.

8. Haykin, S., *Adaptive Filter Theory*, Thomas Kailith, editor, Prentice-Hall, Englewood Cliffs, New Jersey, Chapter 7, 1986.

9. Tufts, D.W., and Kumaresan, R., "Singular Value Decomposition and Improved Frequency Estimation Using Linear Prediction", *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. ASSP-30, pp.671-675, August 1982.

10. Lang, S.W., and McClellan, J.H., "Frequency Estimation with Maximum Entropy Spectral Estimators", *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. ASSP-28, No.6, pp. 716-724, December 1980.

11. Wax, M., and Kailath, T., "Detection of Signals by Information Theoretic Criteria", *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. ASSP-33, No.2, pp. 387-392, April 1985.

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